

# Closed timelike curves in superfluid $^3\text{He}$

Pedro F. González-Díaz and Carmen L. Sigenza  
Instituto de Matemáticas y Física Fundamental  
Consejo Superior de Investigaciones Científicas  
Serrano 121, 28006 Madrid (SPAIN)

June 15, 1998

## Abstract

It is shown that the curved spacetime induced in a thin film of superfluid  $^3\text{He-A}$  by the presence of symmetric vortices with the unbroken symmetry phase, admits the existence of closed timelike curves through which only superfluid clusters formed by anti- $^3\text{He}$  atoms can travel and violate causality.

PACS numbers: 04.20.Gz , 67.57.Fg

Keywords: Rotating vortex, Spacetime metric, Closed timelike curves,

Corresponding Author: P.F. González-Daz, Instituto de Matemáticas y Física Fundamental, C.S.I.C., Serrano 121, 28006-Madrid, Spain

Telephon: (34-1)5616800 Fax: (34-1)5645557

E-Mail: iodpf21@cc.csic.es

One of the most appealing recent developments in physics is the discovery of a common boundary between condensed-matter systems at low temperatures and some cosmological and gravitational systems [1,2]. It has been theoretically established [3] and experimentally checked [4] that the vortices which form when  $^3\text{He-A}$  undergoes the phase transition to its superfluid phase [5] can be regarded as the condensed matter analogs of the so called cosmic strings (i.e. the gauge-theory topological defects which are thought to have been created through phase transitions in the early stages of the evolution of the universe and that could well have been the seeds for presently observed galaxies [6]). On the other hand, it has also been pointed out [7] that event horizons and ergoregions similar to those occurring in black and white holes with angular momentum can be found as well in the curved spacetime of planar solitons moving in superfluid  $^3\text{He-A}$ .

Most symmetric vortices in  $^3\text{He-A}$  can be created in thin films where the unit vector  $\hat{\ell}$  defining the direction of the gap nodes in momentum space is fixed along the normal to the film, and the superfluid circulates around the vortex axis with a velocity given by [8]

$$v_s = \frac{\hbar}{2mr}, \quad (1)$$

with  $m$  the mass of the  $^3\text{He}$  atoms and  $r$  the distance to the vortex axis. In this case, close to their two zeros, the energy spectrum of the Bogoliubov-Nambu fermion quasiparticles becomes that of a charged, massless relativistic particle propagating in a curved spacetime whose cylindrically symmetric metric can be written as [9]:

$$ds^2 = \left(c_{\perp}^2 - v_s^2\right) dt^2 + \left(\frac{\hbar^2}{4m^2c_{\perp}^2} - r^2\right) \left(1 - \frac{v_s^2}{c_{\perp}^2}\right)^{-1} d\phi^2 - \frac{c_{\perp}^2 dz^2}{c_{\parallel}^2} - dr^2 + \frac{\hbar}{m} dt d\phi, \quad (2)$$

where  $c_{\perp} = \frac{\Delta}{p_F}$  and  $c_{\parallel} = v_F = \frac{k_F}{m}$  are the speed of light transverse to and along

$\hat{\ell}$ , respectively, with  $\mathbf{k}_F = P_F \hat{\ell}$  the Fermi momentum and  $\Delta \ll k_F v_F$  the gap amplitude. In metric (2) the true singularity at the horizon  $v_s = c_\perp$  marks the onset of the ergoregion with  $v_s > c_\perp$  and occurs at exactly the radius of the vortex core:

$$r_c = \frac{\hbar}{2mc_\perp}. \quad (3)$$

In this letter we shall consider the possible formation of closed timelike curves (CTCs) in the spacetime described by metric (2). It will be seen that ordinary  $^3\text{He}$  atoms in the superfluid phase cannot enter any of such CTCs and that it is only for  $^3\text{He}$  anti-atoms, which are made of the antiparticles to the particles that form up an  $^3\text{He}$  atom, that the spacetime described by metric (2) can display CTCs. We also discuss the possibility to make observable the effects that such CTCs might have in the neighbourhood of the transition temperature to the superfluid phase.

We first note that the line element (2) becomes the spacetime metric of a spinning cosmic string [10] at points sufficiently far from the vortex axis,  $r \gg r_c$ , only if we interpret the quantity  $\hbar/8mG$  as the internal angular momentum  $J$  per unit length in the resulting spinning string and adscribe to this a zero mass per unit length,  $\mu = 0$ . Far from the vortex axis, metric (2) reduces to

$$ds^2 = c_\perp^2 dt^2 + \left( \frac{\hbar^2}{4m^2 c_\perp^2} - r^2 \right) d\phi^2 - \frac{c_\perp^2 dz^2}{c_\parallel^2} - dr^2 + \frac{\hbar}{m} dt d\phi. \quad (4)$$

Much as for the cylindrically symmetric metric that describes the spacetime of a homogeneously rotating Gdel universe [11], at first glance, one could expect that CTCs would be formed in some regions of the spacetime described by metric (4). However, in order for the Killing vector  $\partial_\phi$  with closed orbits to be timelike everywhere and so allow for CTCs, the coordinate  $r$  must be constrained to the

range  $r < r_c$ , a condition that clearly contradicts the approximation where metric (4) is valid.

For the more general metric (2) one would in principle not expect CTCs to appear, too, neither outside nor inside the horizon at  $r_c$ , the reason for the latter case being that the condition  $r < r_c$  necessarily implies  $v_s > c_\perp$ , with  $v_s = c_\perp$  at  $r = r_c$ , and hence the Killing vector  $\partial_\phi$  keeps being spacelike even on the region inside the vortex. Let us consider however the line  $L_0$  defined by  $L_0 : t = +\beta\phi, r = R, z = 0$ . For that line the  $g_{\phi\phi}$ -component of the metric tensor becomes:

$$c_\perp^{-2}g_{\phi\phi} = \frac{\hbar^2}{4m^2c_\perp^2} - r^2 + \frac{\hbar\beta}{mc_\perp^2} (c_\perp^2 - v_s^2). \quad (5)$$

Thus,  $\partial_\phi$  can still be timelike, provided that

$$r_c < r = R < r_0 = \sqrt{\frac{\hbar\beta}{m}}, \quad (6)$$

with

$$\beta > \frac{\hbar}{4mc_\perp^2} > 0. \quad (7)$$

We note now that any  ${}^3\text{He}$  atoms propagating along any line satisfying condition (6), (7) forward in time can also be described as the corresponding anti- ${}^3\text{He}$  atoms (i.e. atoms that are made of the antiparticles to the particles which an  ${}^3\text{He}$  atom is made of) moving backward in time on that line. Thus, one can always consider another line in the spacetime described by metric (2), defined as  $L_1 : t = -\alpha\phi, r = R, z = 0$ , along which anti- ${}^3\text{He}$  atoms (with negative mass  $-m$ ) propagate backward in time. If we set  $\alpha$  and  $\beta$  to be both positive and satisfying  $\alpha > \beta$ , then line  $L_1$  will also be timelike everywhere and, since an initial point  $Q(\phi = 0)$  and a final point  $P(\phi = 2\pi)$  on  $L_1$  will also be on  $L_0$ , where  $P$  precedes  $Q$ , one can readily deduce that, relative to a given observer that evolves

forward in time, CTCs are actually allowed to occur in the exterior superfluid region  $r > r_c$  of the spacetime described by metric (2), provided that superfluid  $^3\text{He}$  can travel through such CTCs only in the form of anti-atoms.

Nevertheless, the probability for these CTCs to exist and carry with superfluid clusters of atoms through them would not depend on the presence of some nonzero proportion of anti- $^3\text{He}$  or any pair creation process originally in the sample. Relative to a given observer evolving forward in time, even when no initial trace of anti- $^3\text{He}$  existed, the above CTCs had to exist and be operative, since ordinary  $^3\text{He}$  atoms traveling through them would behave like their corresponding anti-atoms to the given observer.

Finally, we briefly comment on the possibility of an experimental verification of these CTCs. This would naturally reduce to the question, how could superfluid anti- $^3\text{He}$  be detected in an experiment with normal superfluid  $^3\text{He}$ ? In order to try to answer this question, let us suppose that anti- $^3\text{He}$  is traveling through a CTC into the past. Of course, it can do so by time amounts which depend on the value of  $\beta$  and, therefore, such a traveling can be so large as for making an observer able to discriminate the presence of superfluid anti- $^3\text{He}$  in supercooling experiments on thin films containing ultrapure (without any trace of  $^4\text{He}$ )  $^3\text{He}$ , before this has reached the superfluid transition temperature  $T_c$ . If superfluid anti- $^3\text{He}$  can travel into the past the way we have discussed in this letter, one could expect that before reaching  $T_c$  clusters of some tens of anti- $^3\text{He}$  atoms able to show superfluid behaviour would momentarily appear still in the viscous  $^3\text{He}$  phase, even though the sample did not contain any trace of  $^4\text{He}$ .

A way to try to detect these superfluid anti- $^3\text{He}$  clusters would require dissolving a given proportion of a suitable small molecule  $A$  in the liquid helium,

checking whether the added molecules are able to freely rotate in short time intervals by the use of ultrafast laser spectroscopy, in an experiment similar to that recently performed by Grebenev, Toennies and Vilesov to determine the minimum number of helium atoms needed for superfluidity [12]. However, expectation to detect CTCs in this way would be very small because viscous  $^3\text{He}$  should attract the molecules much more strongly than anti- $^3\text{He}$  would do. This expectation very much increased if, instead of molecule A, we were able to use the corresponding anti-molecule around which anti- $^3\text{He}$  atoms could much easierly cluster.

Although superfluid liquid helium acts as a true vacuum with respect to its viscous phase, experiments like the one discussed above would also be useful to check violation of causality because of the presence of an observer who could always decide to stop and invert the supercooling process before it reached the transition temperature  $T_c$ , while still detecting causally-disconnected clusters of superfluid anti- $^3\text{He}$  in the viscous liquid.

**Acknowledgements.** The authors acknowledge DGICYT by support under Research Project No. PB94-0107, and A. Ferrera, L.J. Garay and G.A. Mena Marugn for useful comments.

## References

- 1** W.H. Zurek, Phys. Rep. 276 (1997) 177.
- 2** G.E. Volovik and T. Vachaspati, Int. J. Mod. Phys. B10, (1996) 471.  
W.H. Zurek, Nature 317 (1985) 505.
- 4** P.C. Hendry, N.S. Lawson, R.A.M. Lee, P.V.E. McClintock and C.H.D. Williams, Nature 386 (1994) 315; C. Buerle, Yu.M. Bunkov, S.N. Fisher, H. Godfrin and G.R. Pickett, Nature 382 (1996) 332; V.M.H. Ruutu, V.B. Eltsov, A.J. Gill, T.W.B. Kibble, M. Krusius, Yu.G. Makhlin, B. Placais, G.E. Volovik and W. Su, Nature 382 (1996) 334.
- 5** A.J. Leggett and S. Yip, in: *Helium Three*, eds. W.P. Halperin and L.P. Pitaevskii (Elsevier Science Publishers, B.V., 1990).
- 6** N. Turok and R.H. Brandenberger, Phys. Rev. D33 (1986) 2175.
- 7** T.A. Jacobson and G.E. Volovik, *Event horizons and ergoregions in  $^3\text{He}$* , cond-mat/9801308; W.G. Unruh, Phys. Rev. Lett. 46 (1981) 1351; D51 (1995) 2827.
- 8** G.E. Volovik, *Superfluid  $^3\text{He}$ , Particle Physics and Cosmology*, cond-mat/9711031.
- 9** G.E. Volovik, Pis'ma ZhETF 62 (1995) 58 [Engl. transl.: JETP Lett. 62 (1995) 65].
- 10** D. Harari and A.P. Polychronakos, Phys. Rev. D38 (1988) 3320.
- 11** K. Gdel, Rev. Mod. Phys. 21 (1949) 447.
- 12** S. Grebenev, J.P. Toennies and A. Vilesov, Science 279 (1998) 2083.